# Fast Calculation of Hydrodynamic Interaction among Particles

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## Abstract: [1]

The aim of this work is to present an efficient computational method of hydrodynamic interaction among spherical particles in low-Reynolds-number flows. Stokesian Dynamics[1)] has two major difficulties: the heavy computational load scaled by $O(N^3)$, and no systematic way to improve approximation. These difficulties will overcome by multipole expansion, iterative scheme[2,3)], and fast multipole method[4)].

In the multipole expansion for velocity field, we use simple derivatives in Cartesian coordinate rather than harmonic functions in spherical coordinate. This gives us a systematic improvement on accuracy as a generalized grand mobility problem which relates between velocity moments and force moments.

For the practical situation, such as mobility problem for rigid particles where lower orders of irreducible force moments are given and higher orders of irreducible velocity moments should be vanished, we need to solve linear set of equations with generalized grand mobility matrix. In terms of iterative method for linear set of equations, we could solve the problem with $O(N^2)$ where $N$ is the number of particles. The iterative method is constructed only by the calculation of dot-product between the mobility matrix and force moments and that calculation becomes a bottleneck in this case.

Fast multipole method which is applicable to calculate velocity moments with force moments, that is, generalized grand mobility problem, and this gives us further optimization of the calculation. At best, $O(N)$ scheme is possible.

The preliminary calculations and hopefully some demonstrations of concrete problems would be given on the poster.

[1]References:

1) J. F. Brady & G. Bossis, Annu. Rev. Fluid Mech. **20**, 111 (1988).
2) K. Ichiki & J. F. Brady, submitted to Phys. Fluids.
3) R. Weiss, *Parameter-Free Iterative Linear Solvers* (Akademie Verlag, Berlin, 1996).
4) L. Greengard and V. Rokhlin, J. Comput. Phys. **73**, 325 (1987).