Closure relations of non-uniform suspensions

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This paper presents numerical and theoretical results on the study of non-uniform suspensions. One of the most important problems of suspensions is to derive the governing equation on the higher level of two fluid models from the lower level of particle dynamics. There are a lot of works on this context, however, people usually tries to create a model intuitively, or phenomenologically. This is because of lack of the connection between the levels. The present work

- takes into account non-uniformity, and
- derives the constitutive relation from detailed numerical simulations of particles.

Using numerical simulations of particles under Stokes approximation [1], we solve three types of problems – sedimentation problem, the problems of applying torque, and shear flows – under the periodic boundary condition. From the simulations, we can evaluate any quantities – such as translational and angular velocities of particle, mixture pressure and velocity, and viscous stress [2].

The ensemble is random hard sphere configurations, so that the direct results of the ensemble is on the uniform suspensions. However, the ensemble contains any kinds of configurations, in principle, so that we can generate non-uniform ensemble introducing probability weight. In this paper, we study the simplest non-uniformity where the number density is

$$n(\boldsymbol{x}) = n_0 \left(1 + \epsilon \sin \boldsymbol{k} \cdot \boldsymbol{x}\right). \quad (1)$$

We assume here that the parameter ϵ is small, and neglect higher orders of $O(\epsilon^2)$ [3].

It is found that there are couplings among three problems for non-uniform suspensions, while they are decoupled for uniform suspensions. In fact, for uniform suspensions, important quantities are sedimentation velocity, angular velocity, and viscous stress for sedimentation, torque, and shear problems respectively (and separately). For non-uniform suspensions, on the other hand, viscous stress has contributions also on sedimentation and torque problems.

The simple equations such as sedimentation velocity for sedimentation problem, angular velocity for torque problem, and viscous stress for shear problem is not general for three problems. However, the results of non-uniform suspensions suggest that there would be general relations for all three problems, and, therefore, they are the constitutive relation for suspensions.

To derive the constitutive relations, we construct closure relations using all appropriate quantities and close them using numerical results [4]. For example, the closure relation of the symmetric part of viscous stress S is

$$\mathsf{S} = 2\mu_e \mathsf{E}_m + 2\mu_\Delta \mathsf{E}_\Delta + 2\mu_\nabla \mathsf{E}_\nabla, \quad (2)$$

where E_m is the strain of the mixture, E_Δ is the strain related to the slip velocity between the particles and the medium, and E_∇ is the strain related to the gradient of the volume fraction of the particles. We cannot create other symmetric tensor, so that it is the most general relation to close. Note that we only have the term of μ_e for only shear flows on uniform suspensions. Each quantities above are parameterized by basic quantities such as wave vector of non-uniformity $\hat{\boldsymbol{k}}$, gravity $\hat{\boldsymbol{g}}$ for sedimentation, torque $\hat{\boldsymbol{T}}$ for torque, and strain Γ for shear problems. This closure procedure gives equations for coefficients μ_e, μ_Δ, and μ_∇.

The above closure relation is, in fact, independent of the type of the three problems, so that it is the constitutive relation. We also study the anti-symmetric part of the viscous stress in the same manner.

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